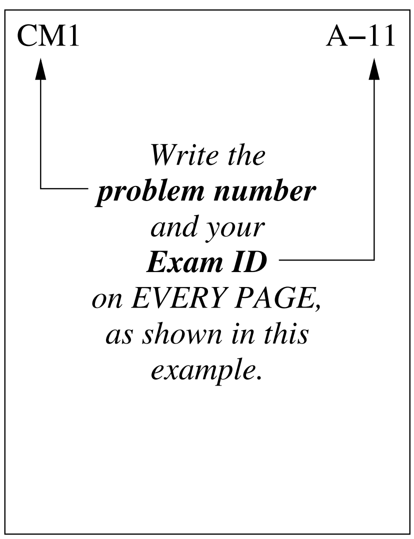
Department of Physics
Montana State University

Qualifying Exam
January, 2026

Day 1
Electricity and Magnetism



- Show your work.
- Write your solutions on the blank paper that is provided.
- Begin each problem on a new page. Write on only one side.
- If you do not attempt a problem, please turn in a blank sheet with your Exam ID and the problem number.
- Turn your work in to the proctor. There is a stack for each problem.
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(EM1) A point dipole with electric dipole moment $\mathbf{p} = p\,\hat{\mathbf{z}}$ is placed at the center of an **isolated, uncharged, conducting** spherical shell of inner radius a and thickness d. You are given several possibly non-zero terms of the potential inside the sphere (all higher order terms will be zero)

$$\varphi(r,\theta) = A + \frac{B}{r} + \left(Cr + \frac{D}{r^2}\right)\cos\theta$$

where (r,θ) are radius and polar angle in spherical coordinates. Use the boundary conditions, and $r \to 0$ asymptotic behavior to determine all coefficients in this expression. From it, find the distribution of induced charges on the sphere. Sketch the electric field inside and outside the shell.

Gradient in spherical coordinates is

$$\boldsymbol{\nabla} = \hat{\boldsymbol{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

(EM2) A charge-neutral, solid cylinder has radius R and height h. The material composing it obeys Ohm's Law and has uniform electrical conductivity σ. Within the cylinder is a uniform, time-dependent magnetic field $\boldsymbol{B}(t)$ directed along its axis.

(a) Calculate the *total rate* at which energy is dissipated in the cylinder due to its finite resistivity in the presence of a time-varying magnetic field.

(b) Describe a scenario whereby the experimentalist provides the energy that is dissipated in the material.

(EM3) A parallel plate capacitor is made of two conducting plates of area A, separated by $d \ll \sqrt{A}$. The capacitor is connected to a battery of voltage V_0, and allowed to fully charge. Then a dielectric slab of thickness $d/2$ and dielectric constant $\epsilon = 2\epsilon_0$ is inserted into the capacitor. This experiment is repeated in two different ways.

(a) In the first experiment, the battery is first **disconnected** and **then** the dielectric slab is inserted. Find the charge Q on the plates **and** the total electric field energy W_E, after the slab has been inserted.

(b) In the second experiment, the battery is left connected while the dielectric slab is inserted. Find Q and W_E for this case.

(c) In which configuration, (a) or (b), does the capacitor store more charge; in which configuration, does it store more electric field energy? Explain why.

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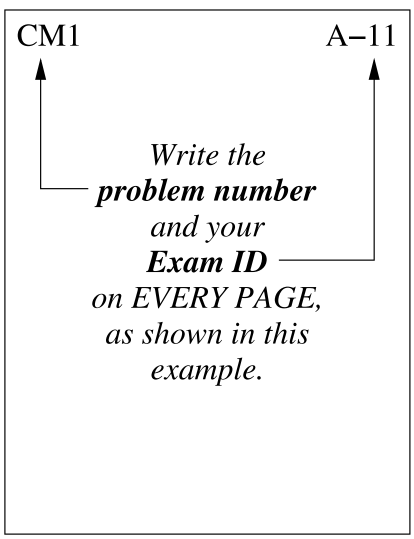
Day 2
Statistical and Thermal Physics



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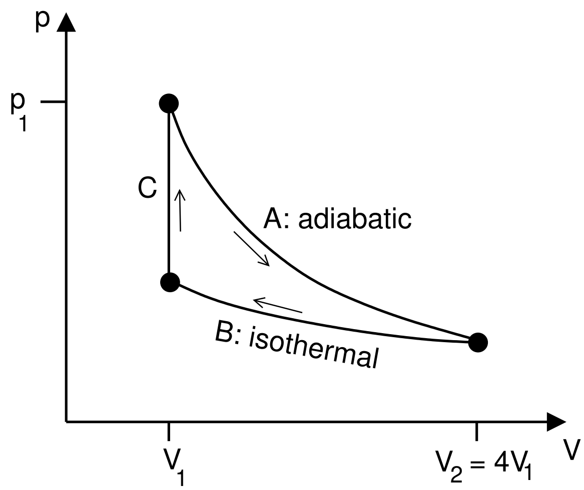
(ST1) A piston is filled with an ideal gas whose adiabatic index is $\gamma = 3/2$. The gas goes through a cycle composed of three stages, A, B, and C:

stage A: Beginning at pressure p_1 and volume V_1 the gas expands **adiabatically** to a volume $V_2 = 4V_1$.

stage B: The gas is **isothermally** compressed back to volume V_1.

stage C: The gas remains at volume V_1 while it returns to pressure p_1



Answer the following question about the cycle.

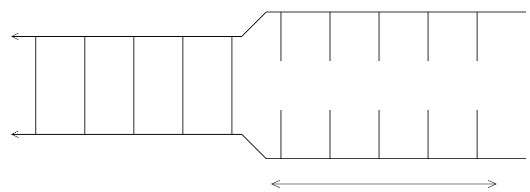
(a) Compute the total work **done by the gas** over the complete cycle. Express you result in terms of p_1 and V_1, being careful to assign the correct sign.

(b) Draw a graph of the cycle in S–T coordinates, with S as the horizontal axis. Label stages A, B, and C. Identify in which stage(s), if any, heat is **expelled from the gas**.

(c) Compute the heat **expelled** from the gas in stages identified in part (b). Express your result in terms of p_1 and V_1 **only**. You may wish to use the expression for the entropy of N particles of an ideal gas (up to an additive constant).

$$S = \frac{k_{\mathrm{B}} N}{\gamma - 1} \ln\big(p V^{\gamma}\big) \quad . \tag{1}$$

(ST2) An infinitely-long strand of DNA is in equilibrium with its environment at temperature T. The strand can unzip to some extent by first breaking a bond, of binding energy A, at one end. The strand can unzip further by breaking the next bond (also with binding energy A), and so on down the DNA strand. The strand will have, in general, N consecutive broken bonds counting from the right end of the strand, as shown:



(a) Determine the average number of bonds $\langle N \rangle$ that will be broken from the one end, and sketch this versus inverse temperature $\beta \equiv 1/k_B T$. Under what conditions does $\langle N \rangle$ become much larger than one?

(b) Determine the mean value of the size of the fluctuations in N about $\langle N \rangle$, in terms of βA alone. Describe how the size of the fluctuations, relative to $\langle N \rangle$, varies with temperature.

(ST3) Consider a degenerate Fermi gas at zero temperature. The gas has N non-relativistic, non-interacting particles (each with mass m) and occupies a volume V. (The shape of this volume does not matter.) The degeneracy of each quantum state is g_s and is treated as a known constant.

(a) Find the Fermi energy ϵ_F of the gas in terms of its number density $n = N/V$. The Fermi energy is the energy of the highest occupied single-particle state when the gas is in its ground state.

(b) Find E_t, the total energy of the system in its ground state, and from it the average energy per particle E_t/N. How does E_t/N scale with the volume V?

(c) Find the pressure P of the ground state, in terms of n.

(d) Now consider a separate volume of a classical ideal gas with the same number density. What is the temperature required for the ideal gas to have the same pressure as the Fermi gas?