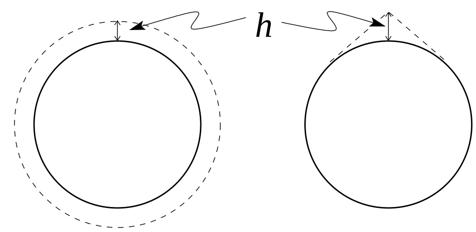
Camel through the needle's eye



For this problem, it should be interesting to compare your intuition with calculations.

A rope is tightly wound along Earth's equator. One adds $\ell = 1$ meter to its length and uniformly expands it away from the surface. To what height h will the rope rise above Earth's surface? Which animal do you think will be able to go ander it: an ant, a cat, a camel, an elephant?

Next, instead of 1 meter one adds $\ell = 5$ mm to the initial rope length, but now pinches one point and pulls it away from the surface. To what height this point will rise? Would one be able to squeeze the animals under it now?

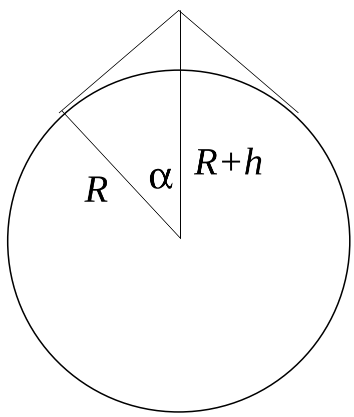
Find an (approximate) analytic expression for $h(\ell)$ for both cases.

Answer of problem **Camel through the needle's eye**

(a) For the uniform expansion the new height is independent of Earth's radius:

$$\boxed{h = \frac{\ell}{2\pi}}$$

For one added meter that corresponds to 16 cm in height, and a cat can crawl under it easily.



(b) From the geometry we find angle α which we assume is small,

$$R\alpha + \ell/2 = R\tan\alpha \approx R\left(\alpha + \frac{1}{3}\alpha^3\right) \qquad \Rightarrow \qquad \alpha \approx \left(\frac{3\ell}{2R}\right)^{1/3}$$

Note that although ℓ/R can be minute, the angle is power 1/3 of that and could be quite significant (but still much less than 1 radian, and the correction to this value is order $O(\ell/R)$ - very small!). The point will rise to height,

$$h = \frac{R}{\cos\alpha} - R \approx R\frac{1}{2}\alpha^2 \qquad \Rightarrow \qquad \boxed{h = \frac{R}{2}\left(\frac{3\ell}{2R}\right)^{2/3}}$$

(As an exercise go and find next order correction to this result, and show that it can be safely neglected.)

For the Earth with $R \approx 6400$km and $\ell = 5$mm, we have $h \approx 3.5$m, so a camel can easily go under it, and even an elephant may have enough room.