N by NW

You are driving at a constant speed of $v = 30\ m/s$, always in the NW direction. You are driving on a horizontal sheet of ice on the Arctic Ocean, with coefficient of friction $\mu = 0.1$. At what distance R from the N pole do you start to skid? Take $g = 9.8\ m/s^2$.

Answer of problem **N by NW**

You start to skid when the required force to accelerate along your path is equal to the maximum possible friction force:

$$m|\ddot{\mathbf{r}}| = \mu m g \tag{1}$$

As we'll see the point of skidding is quite close to the pole, so we can neglect the Earth's curvature and assume we are on a flat surface of a skate rink. Let's work in cylindrical coordinates with origin at the pole, radius r and angle ϕ. The radius-vector is $\mathbf{r} = r\hat{r}$ and we have to remember that the unit vectors in cylindrical coordinates are function of angle: $\hat{r} = \hat{r}(\phi)$ and $\hat{\phi} = \hat{\phi}(\phi)$. This means we have to differentiate them as well, when finding the acceleration. Velocity is

$$\dot{\mathbf{r}} = \dot{r}\hat{r} + r\dot{\hat{r}} = \dot{r}\hat{r} + r\frac{d\hat{r}}{d\phi}\dot{\phi} = \dot{r}\hat{r} + r\hat{\phi}\dot{\phi}$$

and since the velocity is always pointing in NW direction (45 degrees to both parallels and meridians) we can write

$$\dot{r} = -\frac{v}{\sqrt{2}} \qquad\qquad r\dot{\phi} = -\frac{v}{\sqrt{2}}$$

For acceleration we find

$$\ddot{\mathbf{r}} = \frac{d}{dt}[\dot{r}\hat{r} + r\dot{\phi}\hat{\phi}] = [\ddot{r} - r\dot{\phi}^2]\hat{r} + [\dot{r}\dot{\phi} + \frac{d}{dt}(r\dot{\phi})]\hat{\phi}$$

Using the values of velocity components and their time-invariance, we get

$$\ddot{\mathbf{r}} = -r\dot{\phi}^2\hat{r} + \dot{r}\dot{\phi}\hat{\phi} = -\frac{v^2}{2r}\hat{r} - \frac{v^2}{2r}\hat{\phi} \qquad \Rightarrow \qquad |\ddot{r}| = \frac{v^2}{2r}\sqrt{2}$$

which means that the distance where skidding starts is

$$\boxed{R = \frac{v^2}{\sqrt{2}\mu g} = 649\ m\,.}$$