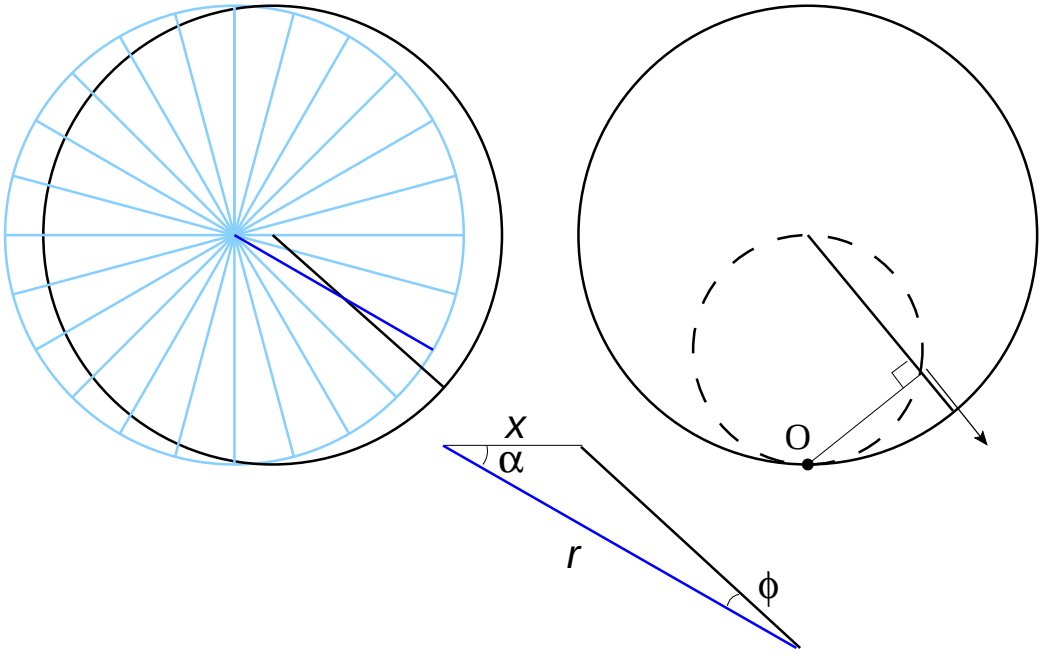
Spoke caustic

A wheel with spokes rolls on the ground. A stationary camera takes a picture of the wheel. Due to the nonzero exposure time of the camera, the spokes will generally appear blurred. At what location(s) in the picture does (do) the spoke(s) not appear blurred?

Note that 'not blurred' means that at some position(s) the spoke element seem not moving, but does not mean that the atoms of the spokes are not moving.

Answer of problem **Spoke caustic**



As the wheel rolls to the right, it rotates clockwise. A spoke shifts to the right by distance x and rotates by angle ϕ that are related

$$\phi = \frac{x}{R}$$

where R is the radius of the wheel. We want to find the points of the wheel that does not seem to move. This is the intersection points of the spoke with itself at a slightly later time ($\phi \to 0$), from the Law of Sines:

$$\frac{x}{\sin\phi} = \frac{r}{\sin(\pi - \alpha - \phi)} \qquad \Rightarrow \qquad r(\alpha) = \frac{R\phi}{\sin\phi}\sin(\alpha + \phi) = \boxed{R\sin\alpha}$$

that represents a circle of radius $R/2$ centered between the center of the wheel and the instanteneous rotation point O.

Solution 2: if you think about point of instanteneous rotation O, all the elements of the wheel are circling around it. The spoke elements that seem stationary are those that are under this rotation shift along the spoke itself! This means (figure on the right) that the angle between the spoke and the line that connects O and the element of the spoke must be 90 degrees. The geometrical figure that correspond to such a configuration is a circle!