

Steeplechase flea

A tree trunk lies on the ground. The trunk has a shape of a cylinder with radius R . A flea attempts to jump over the trunk. What is the minimum initial speed v that enables the flea to clear the trunk? Assume that the flea is intelligent enough to select the optimal take-off point on the ground.

The flea has to jump as shown in the picture - not touching the top of the tree trunk!

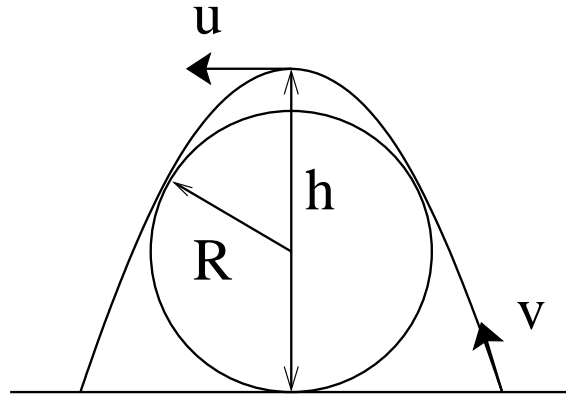
- Take $t = 0$ to be the moment when the flea is at the highest point h . At this point the velocity is horizontal u .

- Find $u(h)$ that allows the flea to skim the surface of the trunk by recognizing that along flea's path the minimal distance from the center of the tree is R .

- Conservation of energy gives ground speed $v_{take-off} = v_{take-off}(u(h), h)$

- Find its minimum as a function of h .

The optimal take-off speed that would allow flea to just graze the trunk at some point:



$$v = \sqrt{2 + \sqrt{8}} \sqrt{gR} \approx 2.197 \sqrt{gR}$$

What is the trajectory that would allow the flea to clear this obstacle fastest?