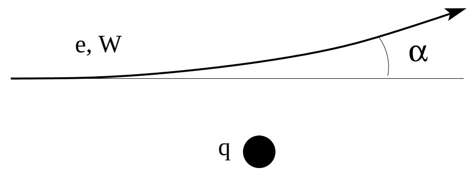
Scattering on a wire



A charged particle with kinetic energy W flies past long uniformly charged wire. The particle moves in the plane perpendicular to the wire, and is deflected by a *small* angle α from initial direction. Find this (small) angle. Particle's charge is e, linear charge density on the wire is q. (Reminder, electric field of the wire is $E = 2q/r$.)

Answer of problem **Scattering on a wire**

$$\boxed{\alpha = \pi \frac{eq}{W}}$$

Since the scattering angle is small, assume as zeroth approximation that the particle moves along a straight line. The closest point of approach (impact parameter) we'll denote h and position on the line will be given by $x = h \tan\phi$ with $-\pi/2 < \phi < \pi/2$.

Again, in zeroth approximation the x-component of velocity is unchanged, and the y-component of velocity is given by

$$m\frac{dv_y}{dt} = F_y = eE(r)\cos\phi = \frac{2eq}{r}\cos\phi = \frac{2eq}{h}\cos^2\phi$$

To eliminate unknown time of flight, use

$$v_x = \frac{dx}{dt} = \frac{h}{\cos^2\phi}\frac{d\phi}{dt}$$

Combining two equations we obtain,

$$m\frac{dv_y}{d\phi} = \frac{2eq}{v_x}$$

As the particle flies by, the angle changes by π and the vertical component of velocity on far right is

$$v_y = \frac{2eq}{mv_x}\pi \qquad \Rightarrow \qquad \alpha \approx \frac{v_y}{v_x} = \frac{\pi eq}{W}$$

- independent of the impact parameter.