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GRAVITATIONAL RADIATION AND ROTATION OF ACCRETING NEUTRON STARS

LARS BILDSTEN
Department of Physics and Department of Astronomy, 366 LeConte Hall, University of California, Berkeley,
Berkeley, CA 94720; bildsten@fire.berkeley.edu
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ABSTRACT

Recent discoveries by the *Rossi X-Ray Timing Explorer* indicate that most of the rapidly accreting ($\dot{M} \gtrsim 10^{-11}\ M_{\odot}\ \mathrm{yr}^{-1}$) weakly magnetic ($B \ll 10^{11}$ G) neutron stars in the Galaxy are rotating at spin frequencies $\nu_s \gtrsim 250$ Hz. Remarkably, they all rotate in a narrow range of frequencies (no more than a factor of 2, with many within 20% of 300 Hz). I suggest that these stars rotate fast enough so that, on average, the angular momentum added by accretion is lost to gravitational radiation. The strong ν_s-dependence of the angular momentum loss rate from gravitational radiation then provides a natural reason for similar spin frequencies. Provided that the interior temperature has a large-scale asymmetry misaligned from the spin axis, then the temperature-sensitive electron captures in the deep crust can provide the quadrupole needed ($\sim 10^{-7}MR^2$) to reach this limiting situation at $\nu_s \approx 300$ Hz. This quadrupole is only present during accretion and makes it difficult to form radio pulsars with $\nu_s > (600\text{–}800)$ Hz by accreting at $\dot{M} \gtrsim 10^{-10}\ M_{\odot}\ \mathrm{yr}^{-1}$. The gravity wave strength is $h_c \sim (0.5\text{–}1) \times 10^{-26}$ from many of these neutron stars and greater than 2×10^{-26} for Sco X-1. Prior knowledge of the position, spin frequency, and orbital periods will allow for deep searches for these periodic signals with gravitational wave interferometers (LIGO, VIRGO, and the "dual-recycled" GEO 600 detector), and experimenters need to take such sources into account. Sco X-1 will most likely be detected first.

Subject headings: accretion, accretion disks — dense matter — gravitation — stars: neutron — stars: rotation — X-rays: bursts

1. INTRODUCTION

The launch of the *Rossi X-Ray Timing Explorer* (*RXTE*) has allowed for the discovery of fast quasi-periodic variability from many rapidly accreting ($\dot{M} \gtrsim 10^{-11}\ M_{\odot}\ \mathrm{yr}^{-1}$) neutron stars. These observations strongly suggest that these neutron stars (NSs) are rapidly rotating, as predicted by those scenarios connecting the millisecond radio pulsars to this accreting population (see Bhattacharya 1995 for an overview). Strohmayer et al. (1996) were the first to detect nearly coherent $\nu_B = 363$ Hz oscillations during type I X-ray bursts from the low accretion rate ($\dot{M} < 10^{-9}\ M_{\odot}\ \mathrm{yr}^{-1}$) NS 4U 1728−34. Pulsations were detected in six of the eight bursts analyzed at that time. In addition, two high-frequency quasi-periodic oscillations (QPOs) were seen in the persistent emission. These changed with accretion rate but maintained a fixed difference frequency of $\nu_d \approx 363$ Hz, identical to the period seen during the bursts. The detection of two drifting QPOs (in the persistent emission) separated by a fixed frequency identical to that seen in the bursts naturally leads to beat frequency models (Strohmayer et al. 1996; Miller, Lamb, & Psaltis 1998). The difference frequency is presumed to be the NS spin frequency ν_s, whereas the upper frequency has different origins in different models (see van der Klis 1998 for a summary). In addition, the temporal behavior of the periodic oscillations both during the rise of the bursts (Strohmayer, Zhang, & Swank 1997b) and in the cooling tails (Strohmayer et al. 1997a) are most easily explained in terms of rotation.

There are six NSs with measured periodicities during type I X-ray bursts (see Table 1). Both the difference frequencies (ν_d) and the burst frequencies (ν_B) are in a narrow range, from 260 to 589 Hz. For two objects (KS 1731−260 and 4U 1636−53), the difference frequencies are one-half the burst values. Which value is ν_s is not resolved. There are also many NSs that accrete at higher rates and are not regular type I X-ray bursters. Many of these objects, notably the "Z" sources, also show drifting QPOs at fixed separation, again with a similarly narrow frequency range (roughly 250–350 Hz). Beat frequency–like models are also applied to these observations so as to infer ν_s. The applicability of such a model is less clear when the difference frequency is not constant (Sco X-1, van der Klis et al. 1997; 4U 1608−52, Mendez et al. 1998).

If accreting matter always arrives with the specific angular momentum of a particle orbiting at the NS radius ($R = 10R_6$ km), then it only takes $\sim 10^7$ yr of accretion at $\dot{M} \approx 10^{-9}\ M_{\odot}\ \mathrm{yr}^{-1}$ for a $M = 1.4M_{1.4}\ M_{\odot}$ NS to reach $\nu_s = 50$ Hz from an initially low frequency. It is thus remarkable that these NSs are all rotating at nearly the same rate. White & Zhang (1997) argued that this similarity arises because these NSs are magnetic and have reached an equilibrium at which the magnetospheric radius equals the corotation radius. This requires an intrinsic relation between their magnetic dipoles μ_b and $\dot{M}$ so that they all reach the same rotational equilibrium (most likely $\mu_b \propto \dot{M}^{1/2}$) and a way of hiding the persistent pulse typically seen from a magnetic accretor.

My alternative explanation for these spin similarities is that gravitational wave (GW) emission has started to play an important role. If the NS has a misaligned quadrupole moment Q, then the strong spin frequency dependence of GW emission defines a critical frequency beyond which accretion can no longer spin-up the star. Such a NS will radiate energy via GWs at the rate $\dot{E} = 32GQ^2\omega^6/5c^5$, where $\omega = 2\pi\nu_s$, and lose angular momentum at the rate $N_{\mathrm{gw}} = \dot{E}/\omega$. Balancing this spin-down torque with the characteristic spin-up torque from time-averaged accretion, $N_a \approx \dot{M}(GMR)^{1/2}$, gives the Q needed so as to make the critical frequency 300 Hz,

$$Q \approx 4.5 \times 10^{37}\ \mathrm{g\ cm^2} \left(\frac{\dot{M}}{10^{-9}\ M_{\odot}\ \mathrm{yr}^{-1}}\right)^{1/2} \left(\frac{300\ \mathrm{Hz}}{\nu_s}\right)^{5/2}, \quad (1)$$

or less than 10^{-7} of the NS moment of inertia, $I \approx$

TABLE 1

RAPID PERIODICITIES DURING TYPE I X-RAY BURSTS

Object Name	ν_B (Hz)	ν_d (Hz)	Flux[a]	$h_c/10^{-27}$	Reference
4U 1702−429	330	...	1.0	1.2	1
4U 1728−34	363	363	2.8	2.0	2
KS 1731−260	524	260 ± 10	0.2–2	2.0	3, 4
Aql X-1	549	...			5
4U 1636−53	581	276 ± 10	4.4	2.8	6, 7
MXB 1743−29	589	...			8

[a] Average 2–10 keV fluxes (in units 10^{-9} ergs cm^{-2} s^{-1}) are from van Paradijs 1995.

REFERENCES—(1) J. Swank 1997, private communication; (2) Strohmayer et al. 1996; (3) Smith, Morgan, & Bradt 1997; (4) Wijnands & van der Klis 1997; (5) Zhang et al. 1998a; (6) Strohmayer et al. 1998; (7) Wijnands et al. 1997; (8) Strohmayer et al. 1997a.

10^{45} g cm^2. The similarities in ν_s may then arise because of the weak dependencies of the critical frequency on Q and $\dot{M}$.

What is the source of the misaligned quadrupole? Wagoner (1984) argued that accreting NSs would get hung-up at spin frequencies where the Chandrasekhar-Friedman-Schutz instability sets in. However, Lindblom (1995) and Lindblom & Mendell (1995) have shown that the star needs to be very near the breakup frequency ($\nu_s \gtrsim$ kHz) for such an instability to occur, even for the core temperatures $T_c = (1\text{–}3) \times 10^8$ K of rapidly accreting NSs (Ayasli & Joss 1982; Brown & Bildsten 1998). The spin frequencies for these NSs are too slow for such an instability.

I present in § 2 a new source for lateral density variations in an accreting NS: electron captures (EC) in the crust. The constant compression of the crust forces nuclei to undergo EC when the electron Fermi energy $E_{\rm F}$ is high enough to make a transition. However, the crust is hot enough in a rapidly accreting NSs to make the EC rates temperature sensitive. Hotter regions then capture at lower pressures, so that the density jump associated with the EC is at a higher altitude in the hotter parts of the crust. Moderate lateral temperature variations then lead to density variations large enough to generate the required Q. This outcome is independent of the particular source of the temperature variations. One possible cause for a T asymmetry relative to the spin axis is a weak magnetic field. I conclude in § 3 by finding the GW signal strength and by estimating detection.

2. ELECTRON CAPTURES IN THE NEUTRON STAR CRUST

These NSs accrete hydrogen- and helium-rich material from their companions, and within days of reaching the surface, this matter is burned to heavy elements. The composition of these ashes is still uncertain but most certainly consists of heavy nuclei, potentially beyond the iron group (Schatz et al. 1997). This material replaces the primordial NS crust and becomes neutron rich by successive EC. Later on, neutron emissions and pycnonuclear reactions occur (Bisnovatyi-Kogan & Chechetkin 1979; Sato 1979; Haensel & Zdunik 1990, hereafter HZ). For the $\dot{M}$'s and residual hydrogen abundance (Taam, Woosley, & Lamb 1996) appropriate for these NSs, the temperature of the accumulating matter and deep crust is $T_8 = T/10^8~{\rm K} \approx (2\text{–}6)$ (Brown & Bildsten 1998).

I confine my discussion in this Letter to the outer crust [before neutron drip at $\rho \approx (4\text{–}6) \times 10^{11}$ g cm^{-3}; HZ], which is held up by degenerate and relativistic electrons. They exert a pressure $p = 1.42 \times 10^{30}$ ergs cm^{-3} $(E_{\rm F}/30~{\rm MeV})^4$ from which the mass of the shell above a particular depth is $M_{\rm cr}(E_{\rm F}) = 4\pi R^2 p/g$, where $g = GM/R^2$, or

$$M_{\rm cr}(E_{\rm F}) \approx 5 \times 10^{-5}~M_\odot \frac{R_6^4}{M_{1.4}} \left(\frac{E_{\rm F}}{30~{\rm MeV}}\right)^4 . \qquad (2)$$

The crust is compressed on a timescale $t_{\rm comp} \equiv p/\dot{m}g$, where $\dot{m}$ is the local accretion rate, so that $E_{\rm F}$ rises with time, eventually leading to nuclear EC. Many (Sato 1979; HZ) presumed that the captures were instantaneous once $E_{\rm F} > E_d$, where E_d is the mass difference between the (A, Z) nucleus and $(A, Z-1)$ nucleus. In reality, the transitions will not occur until the EC lifetime is comparable to $t_{\rm comp}$ (Blaes et al. 1990). For low accretion rates $\dot{M} \sim 10^{-16}~M_\odot~{\rm yr}^{-1}$, Blaes et al. (1990) showed that most captures occur in a thin zone where $E_{\rm F} > E_d$ only slightly. The situation is different for the high $\dot{M}$'s of the bright X-ray sources, as the high crustal temperature leads to most EC occurring out on the thermal tail at a physical location where $E_{\rm F} < E_d$.

I consider a region of the crust consisting of a single nucleus of charge Ze and mass Am_p. For many nuclei, the capture to the ground state of the $(A, Z-1)$ nucleus is highly forbidden and proceeds more slowly than $t_{\rm comp}$. The element is still abundant when $E_{\rm F}$ reaches a value at which a more favorable transition to an excited state of the $(A, Z-1)$ nucleus can occur.[1] For this reason, I estimate the EC rates by using ft values in the range of allowed reactions, $ft = 10^4\text{–}10^6$ s. Since these stars are hot enough for the subthreshold EC to dominate, the transition occurs when $E_{\rm F,\,depl} = E_d - \beta k_B T$, where $\beta \approx (8\text{–}12)$ depends logarithmically on $\dot{m}$, T, and ft (Bildsten & Cumming 1998). A hotter region undergoes the transition at a higher altitude (call s_c the distance down to the transition from where $E_{\rm F} = 1$ MeV, i.e., s increases into the star) compared to a colder region. Figure 1 displays the EC transition layer for a NS accreting at $\dot{M} = 2 \times 10^{-9}~M_\odot~{\rm yr}^{-1}$. As an example I chose the nucleus ^{56}Ca, which captures at $E_d \approx 23$ MeV, where $\rho \approx 2.5 \times 10^{11}$ g cm^{-3}. This is the fourth EC transition that HZ found when starting with ^{56}Fe. Since this is a generic process, I show cases for $ft = 10^4$ s (*dotted line*) and $ft = 10^6$ s (*solid line*) for (from left to right in the upper and lower panels) $T_8 = 6$,4, and 2. The density jump associated with the ECs is evident and, if discontinuous, would be $\Delta\rho/\rho = 2/(Z-2) = 0.11$.

If a transverse temperature gradient is present in the crust, then these ECs generate density asymmetries on the star and a quadrupole with mass $\Delta M \approx 4\pi R^2 \Delta\rho \Delta s_c$. The mass inferred from the $ft = 10^4$ s case for a temperature contrast $\Delta T_8 = 2$ (where the difference in physical depth of the EC is $\Delta s_c \approx 500$ cm) is $\Delta M \approx 10^{-7}~M_\odot$, just what is needed for GWs to be important. More generally, the mass involved is $\Delta M = M_{\rm cr}(\Delta\rho/\rho)(4\Delta E_{\rm F,\,depl}/E_{\rm F})$, or since $\Delta E_{\rm F,\,depl} \approx 10 k_B \Delta T$,

$$\Delta M \approx 5.5 \times 10^{-8}~M_\odot~\Delta T_8 \frac{R_6^4}{M_{1.4}} \left(\frac{E_{\rm F}}{30~{\rm MeV}}\right)^3 \qquad (3)$$

for a single EC layer. Since there are a few more electron

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[1] Since the mass difference of the $(A, Z-1)$ and $(A, Z-2)$ nuclei is smaller than the mass difference of the (A, Z) and $(A, Z-1)$ nuclei, the sequence always consists of two successive captures (HZ). This also blocks the inverse reactions. The larger phase space for the second reaction (as well as the larger number of available excited states with favorable spin-parity) means that the $(A, Z-1)$ nucleus captures electrons more rapidly than the (A, Z) nucleus. Since the first capture is the rate-limiting step, I do not track the intermediate $(A, Z-1)$ nucleus.

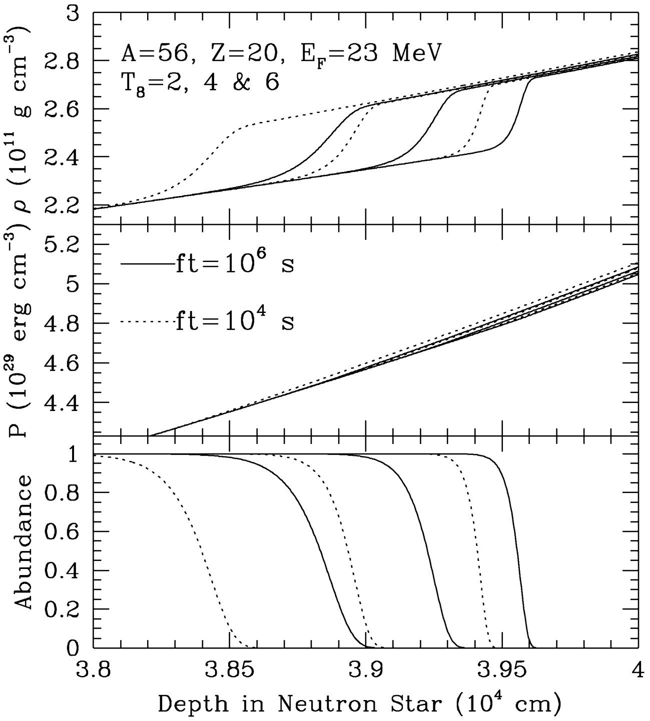

FIG. 1.—Density, pressure, and nuclear abundance in the ^{56}Ca electron capture layer for a $R = 10$ km, $M = 1.4\ M_\odot$ NS accreting at $\dot M = 2 \times 10^{-9}\ M_\odot$ yr^{-1}. These are plotted as a function of increasing depth into the star; deeper regions are to the right. For a fixed value of ft, the hotter crusts deplete sooner. The curves are, from left to right, for $T_8 = 6, 4,$ and 2.

capture layers underneath this one, there is adequate mass to generate the required Q when $\Delta T_8 \approx 1$. The repercussions of the deeper reactions are still unknown.

The ECs deposit heat directly into the crust, generating a flux $F_{\rm deep} = E_{\rm deep}\dot m$, where $E_{\rm deep} \approx 10^{17}$ ergs g^{-1} (HZ). Temporal variations in $\dot m$ lead to local heating at these depths, which takes a few years to equilibrate in the radial direction. The heat transfer is fixed by electron conduction at these depths, so that the conductivity is $K = \pi^2 k_B^2 T n_e c^2/3E_{\rm F}\nu_{e\text{-ph}}$, where $\nu_{e\text{-ph}} \approx 13\alpha k_B T/\hbar$ is the electron-phonon scattering frequency (Yakovlev & Urpin 1980). The time it takes heat to cross a scale height, $H = p/\rho g$, is then $t_{\rm th} \approx \rho C_p H^2/K \sim$ yr, where $C_p \approx 3k_B/Am_p$ is the specific heat at constant pressure.

However, lateral thermal asymmetries will persist as long as the heating time $t_H \approx C_p T y/E_{\rm deep}\dot m \sim 4 \times 10^{-3} T_8 t_{\rm comp}$ is shorter than the time to transport heat around the star at the same depth. That time is $t_{\rm th,\,R} \approx \rho C_p R^2/K \approx 6400$ yr $(E_{\rm F}/30\ {\rm MeV})$, so for $\dot M \gtrsim 3 \times 10^{-11}\ M_\odot$ yr^{-1} $(E_{\rm F}/30\ {\rm MeV})^3(R/10\ {\rm km})^2$, the time to transport heat around the crust is longer than the time to locally heat it. All of the objects that I am discussing are in this regime, and so large-scale temperature asymmetries can persist as long as the system is being perturbed in some way. Although the origin of lateral T gradients is still unknown, it is difficult for conduction to wash them out. For example, large-scale $\dot m$ or compositional variations (due to, say, magnetically channeled flow or a "buried field") will imprint large-scale T asymmetries.

The pressure as a function of depth in the EC region is shown in the middle panel of Figure 1. Even though the density contrasts are large, the pressure contrasts are not. If one part of the star has a slightly deeper capture zone, at $s_{c,2} = s_{c,1} + \Delta s_c$, then the pressures at a fixed depth $s > s_{c,1}$ relate via $P_1 \approx P_2(1 + 8\Delta s_c/Zs)$. The pressure is higher underneath the layer that captured sooner due to the "extra weight" of the dense layer. Since $s \propto E_{\rm F}$, the lateral pressure contrast at a fixed depth, $\Delta P = (P_1 - P_2)$, is $\Delta P/P \approx 80 k_B \Delta T/ZE_{\rm F}$. These transverse pressure gradients would lead to flow and a cancellation of the quadrupole if the matter was in a liquid state. However, as long as $T_8 < 10$, the matter is solid at these depths and has a finite shear modulus, $\mu \approx 10^{-2}p$ (Strohmayer et al. 1991). The transverse pressure gradients are then balanced by a slight amount of shear stress as the matter underneath the cooler regions laterally shifts a distance ξ. Roughly, ξ is found from the transverse momentum equation $\Delta P/R \approx \mu\xi/s^2$, or $\xi/s \approx (100s/R)(8\Delta s_c/Zs)$. This gives $\xi/s \approx 6 \times 10^{-3}\Delta T_8$, or a few meters of transverse displacement over the whole surface. For now, I will presume that this configuration will not crack. Relative vertical motion has yet to be investigated and could reduce the estimated Q.

3. DETECTABILITY OF THE GRAVITATIONAL WAVES

Most accreting, weakly magnetic neutron stars are spinning at $\nu_s \approx (250\text{–}500)$ Hz. I conjecture that this similarity is the result of an equilibrium at which the angular momentum accreted is radiated away in gravitational waves. I showed that prethreshold electron captures will turn any lateral temperature gradients into lateral density gradients. In this case, a small spin-misaligned temperature gradient gives rise to a quadrupole, $Q \sim 10^{-7}MR^2$, that is adequate to explain the similarities at ≈300 Hz. These accretion-induced Q's clearly make it difficult to spin up a neutron star to 1000 Hz by accreting at $\dot M \gtrsim 10^{-10}\ M_\odot$ yr^{-1}. Indeed, an NS spinning at 1000 Hz with $Q \approx 10^{-8}MR^2$ would be difficult to spin-up even at $\dot M \sim 10^{-8}\ M_\odot$ yr^{-1}. The lateral T asymmetries should subside at lower $\dot M$'s, reducing Q (see the dependence in eq. [3]) and potentially making it easier to reach higher ν_s by accreting slowly for a long period of time. There is already some hint of this, since the highest $\dot M$ sources seem to be rotating the slowest.

Independent of the cause of the T asymmetry, if gravitational radiation is the explanation of the spin similarities, then the characteristic GW signal strength at Earth (suitably averaged over spin orientations; Brady et al. 1998) is $h_c = 2.9G\omega^2 Q/dc^4$, where d is the distance to the object (the prefactor is 4 when looking down the spin axis). Presuming that the NS luminosity is $L \approx GM\dot M/R$, then h_c is written in terms of the observable $F = L/4\pi d^2$ (Wagoner 1984),

$$h_c \approx 4 \times 10^{-27}\frac{R_6^{3/4}}{M_{1.4}^{1/4}} \times \left(\frac{F}{10^{-8}\ {\rm ergs\ cm^{-2}\ s^{-1}}}\right)^{1/2}\left(\frac{300\ {\rm Hz}}{\nu_s}\right)^{1/2}, \qquad (4)$$

which represents a lower limit for h_c due to the L-$\dot M$ conversion I chose and since F is never fully measured. The minimum h_c's for those NSs that have ν_s inferred from type I bursts are shown in Table 1. There are many NSs for which only ν_d has been measured (see van der Klis 1998 for a summary). The average 2–10 keV fluxes from van Paradijs (1995) imply the following GW amplitudes: GX 349+2 ($\nu_d = 266 \pm 13$, $h_c \approx 5.4 \times 10^{-27}$; Zhang, Strohmayer, & Swank 1998b), 4U 1820−30 ($\nu_d = 275 \pm 8$, $h_c \approx 3.7 \times 10^{-27}$), GX 17+2 ($\nu_d = 294 \pm 8$, $h_c \approx 4.7 \times 10^{-27}$), 4U 0614+09 ($\nu_d = 327 \pm 4$, $h_c \approx 1.3 \times 10^{-27}$), GX 5−1 ($\nu_d = 327 \pm 11$, $h_c \approx 6.0 \times$

10^{-27}) Cyg X-2 ($\nu_d = 343 \pm 21$, $h_c \approx 3.7 \times 10^{-27}$), and GX 340+0 ($\nu_d = 325 \pm 10$, $h_c \approx 3.7 \times 10^{-27}$; Jonker et al. 1998).

The highest flux object that shows kHz QPOs is Sco X-1, with an orbital period of 18.9 hr. Its difference frequency is not constant, however (van der Klis et al. 1997), and so we are still uncertain about the exact spin frequency. If it is rotating at ≈250 Hz, then $h_c \approx 2.2 \times 10^{-26}$ at $2\nu_s = 500$ Hz. Detection of a weak periodic gravitational wave signal depends on the ability to coherently fold large data sets for a long time, τ, in order to obtain the $\sim (2\nu_s\tau)^{1/2}$ signal enhancement. The initial LIGO interferometer will have $h_{\rm rms} = 10^{-21}$ at $\nu \approx 500$ Hz (Abramovici et al. 1992), requiring integration times of about a year. If the orbital parameters, spin frequency, and phase were known for Sco X-1, then Brady et al.'s (1998) work implies that it would be detected at 99% confidence with the initial LIGO in about 1 yr. However, our lack of detailed prior information on these parameters demands a larger parameter space search. The consequent reduction in the signal-to-noise ratio (maybe a factor of ∼5) make it more likely that detection will have to wait for the enhanced LIGO (see Thorne 1998 for the sensitivities). The similar spin frequencies of so many objects give an advantage to dual-recycling interferometers (such as GEO 600 and TAMA 300). Indeed, the GEO 600 noise strength (K. A. Strain, A. Campbell, J. E. Logan, N. A. Robertson, & S. Rowan 1998, private communication), when tuned to Sco X-1, is competitive with the initial LIGO and VIRGO. Detection of the larger number of NSs[2] with $h = (0.5\text{–}1) \times 10^{-26}$ will have to wait for advanced broadband interferometers (such as advanced LIGO), further developments in the dual-recycled interferometers, or larger cryogenic detectors.

The stability of the GW signal depends on the origin of the mass quadrupole. For the EC origin, I would not expect major differences in Q on timescales shorter than the thermal time (years). Another relevant time is that to replenish the thickness of the EC layer, which is ∼100 yr at $\dot{M} \sim 10^{-9}\ M_\odot\ \mathrm{yr}^{-1}$. However, since $\dot{M}$ varies, we cannot expect the torques to always be in balance. For example, if $\dot{M}$ were to drop suddenly, the instantaneous spin-down of the NS due to the equilibrium Q would be $\dot{\nu} \approx 10^{-13}\ \mathrm{s}^{-2} \left(\langle\dot{M}\rangle/10^{-9}\ M_\odot\ \mathrm{yr}^{-1}\right)$. This would lead to one pulse cycle of drift in a time $\approx(2/\dot{\nu})^{1/2} \approx 50$ days.

How does this work affect the accretion spin-up scenario for making millisecond pulsars? The EC Q will decrease once accretion has halted and the temperature equilibrates. In addition, when the crust is cold, it only takes $10^{-7}\ M_\odot$ of accretion to wash out the compositional gradients generated during accretion at high rates. There is an important comparison to make in this regard. Namely, as long as Q decays on a timescale t_d much shorter than $t_{\text{spin-up}} \equiv I\omega_i/\dot{M}(GMR)^{1/2} \approx 5 \times 10^7$ yr, then the millisecond pulsar is "born" at roughly the same spin period since $\nu_f = \nu_i(1 + 4t_d/t_{\text{spin-up}})^{-1/4}$. If Q persists at some low level long after accretion has ended, it could still play an important role. Indeed, as many have shown (Thorne 1987; New et al. 1995; Brady et al. 1998), a small Q can go a long way toward explaining the observed spin-down of millisecond pulsars *and* making them detectable as GW sources.

Transverse T gradients may be more prevalent on the $B \gtrsim 10^{11}$ G accreting pulsars. However, most of them have $\nu_s \ll$ Hz so that the Q needed for GW equilibrium is implausibly large, making it unlikely that GWs play an important role in their spin evolution. However, there is a lack of accreting pulsars with $\nu_s >$ Hz that is usually explained by stating that all pulsars have reached their magnetic equilibrium and that few have $B \ll 10^{11}$ G. If we presume a maximum allowed quadrupole $Q_{\max} = 10^{-5}MR^2$ (Thorne 1987), then GW emission could play a role when $\nu_s > 35\ \mathrm{Hz}\ (\dot{M}/10^{-9}\ M_\odot\ \mathrm{yr}^{-1})^{1/5}$, only somewhat alleviating this problem.

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[2] The recent *RXTE* discovery of a persistent 401 Hz accreting pulsar (Wijnands & van der Klis 1998) in the transient binary SAX J1808.4-3658 (in't Zand et al. 1998) provides hope for GW studies. It was observed in outburst by *RXTE* (Marshall 1998), and its orbit was measured by Chakrabarty & Morgan (1998). Enough is known about this weakly magnetic ($B < 10^9$ G) accreting pulsar that a coherent data fold can be undertaken. Unfortunately, the time-averaged flux (I estimate 2×10^{-10} ergs cm^{-2} s^{-1} at most) is low and even if looking right down the spin-axis, $h < 7 \times 10^{-28}$. It might be that spin-down in quiescence is the best indicator of GW emission from this NS.

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Note added in proof.—There was a burst of theoretical activity on another gravitational radiation mechanism while this paper was in progress. N. Andersson (ApJ, in press [1998]) recently showed that gravitational radiation tends to excite the *r*-modes of a rotating star. This was confirmed by J. L. Friedman & S. M. Morsink (Phys. Rev. Lett., in press [1998]). Whether this leads to an unstable growth of the *r*-mode depends on a competition with the viscous damping mechanisms.

L. Lindblom, B. J. Owen, & S. M. Morsink (Phys. Rev. Lett., in press [1998]) showed that the modes were unstable for young, hot ($T_c > 10^9$ K) and rapidly rotating neutron stars. Depending on just how nonlinear the mode becomes, this can lead to rapid spin-down (within a year) of neutron stars that are born very rapidly rotating.

These results may also apply to the accreting neutron stars. Using the work of B. J. Owen et al. (Phys. Rev. D, in press [1998]), I estimate that the required amplitude of the $l = m = 2$ *r*-mode so as to radiate angular momentum at the rate supplied by accretion is only 20 cm. This points to the concept that *if the r-modes are unstable in the accreting objects*, then they can possibly explain the spin frequency similarities I have noted. Since the required amplitude is small, it might also be the case that the star gets continuously spun-up by accretion until it hits the instability, after which it would just hover near the stability/instability transition line. In this case, the spin frequency would be set by the core temperature and internal viscous mechanisms, which, although uncertain, would not be expected to vary much across this accreting population. The core temperatures for the accreting neutron stars are (depending on the internal neutrino cooling mechanisms; Brown & Bildsten 1998) in the range $T_c = (1\text{–}4) \times 10^8$ K, where the viscosity mechanisms are uncertain and depend on whether the neutrons are superfluid or normal. The current lower limits to the critical spin frequencies are 100–200 Hz (Lindblom et al. 1998) for these temperatures and will increase when further viscous mechanisms are included. More theoretical work is needed before we can say with certainty whether the *r*-modes can play a role in the accreting neutron stars.