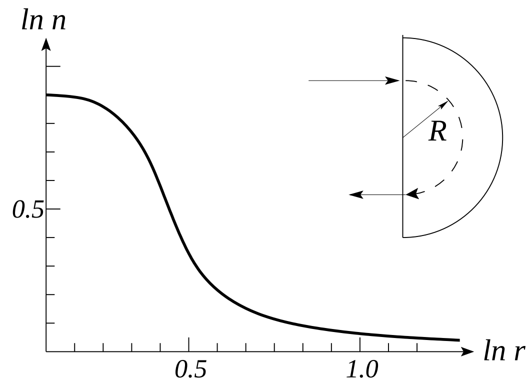
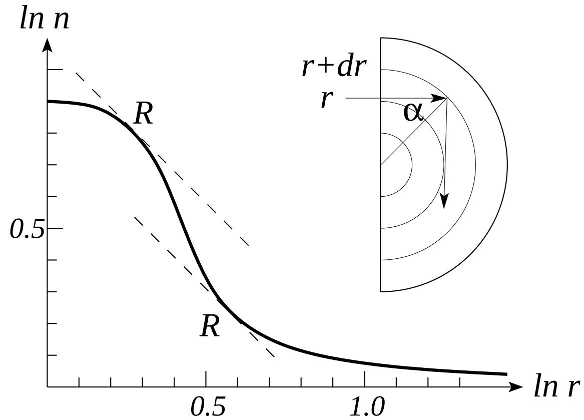
Circular optical channel

Monochromatic light is incident on the flat part of a semi-cylinder made of concentric rings with different refraction coefficients. The dependence of the refraction coefficient on radius is shown on $\ln n - \ln r$ plot.

Find the radii where a narrow beam of light can propagate along exact semicircle and exit in the opposite direction of where it came from.



Answer of problem **Circular optical channel**



One can use Fermat's principle and minimize the time it takes light to go in a circle, $r(\phi) = R = const$

$$t = \int_0^\pi d\phi \frac{n(r(\phi))\sqrt{r'(\phi)^2 + r(\phi)^2}}{c} \ ,$$

or one can imagine an onion with different layers having different refraction coefficients. The light entering at radius r should undergo total internal reflection from the next layer at $r + dr$:

$$\sin\alpha = \frac{r}{r+dr} = \frac{n(r+dr)}{n(r)}$$

and to first order in dr we have,

$$1 - \frac{dr}{r} = 1 + \frac{n'(r)dr}{n(r)}$$

Since dr is arbitrary we have equation for semi-circular optical guides

$$\frac{1}{r} + \frac{1}{n(r)}\frac{dn(r)}{dr}\Big|_R = 0 \qquad \Rightarrow \qquad \frac{r}{n(r)}\frac{dn(r)}{dr} = \boxed{\frac{d\ln n(r)}{d\ln r}\Big|_R = -1}$$

i.e. the radii where light can propagate in a semi-circle correspond to points on the $\ln n - \ln r$ curve with slope -1. There are two such radii, found from the plot graphically, $\ln R_1 \approx 0.3$ and, $\ln R_2 \approx 0.6$.