Gas and Spring

Find the heat capacity of the system consisting of a container that has two compartments separated by a piston. To the right of the piston is a gas of monoatomic molecules (gas parameters $p_0, T_0, V_0$), and to the left is vacuum. The piston is held by a spring. If the gas is removed, the piston touches the right wall and the spring is relaxed.

Heat capacities of the materials composing spring, piston and container walls can be neglected.
Answer of problem  

Gas and Spring

Answer:

\[ C = \frac{dQ}{dT} = \frac{dU_{gas}}{dT} + \frac{dU_{spring}}{dT} \]

- the heat supplied to the system goes into energy of the gas and energy of the spring (gas does work on the spring, but this work is stored in spring inside the system, so we don’t have to include it twice; one can also think about the entire system doing no work on its surroundings and only the internal energy of the system changing, which includes energy of the gas and spring).

Force of the spring compressed by \( x \) balances pressure

\[ pS = kx \quad pV = pSx = kx^2 = \nu RT \]

and using this connection between \( x \) and \( T \) we get

\[ C = \frac{3}{2} \nu R + kx \frac{dx}{dT} = \frac{3}{2} \nu R + kx \frac{1}{2kx} \nu R = 2\nu R \]

\[ C = 2\frac{p_0 V_0}{T_0} \]