On the $xy$ plane determine region(s) where

$$x^y > y^x$$

and find which is greater, $e^\pi$ or $\pi^e$?

Thanks to Recep Avci for suggesting this problem.
Answer of problem \[ e^\pi \ ? \ \pi^e \]

Take log of both sides, and determine when

\[
\frac{\ln(x)}{x} = \frac{\ln(y)}{y}.
\]

This equation has two solutions,

\[ y = x \]

and another one that exists in \( x > 1, \ y > 1 \) region, and looks like a hyperbola, symmetric about \( x = y \) line.

One can determine that

\[ e^\pi > \pi^e \]