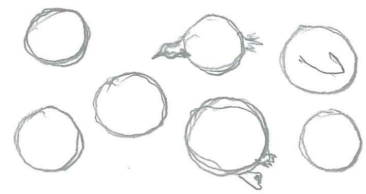
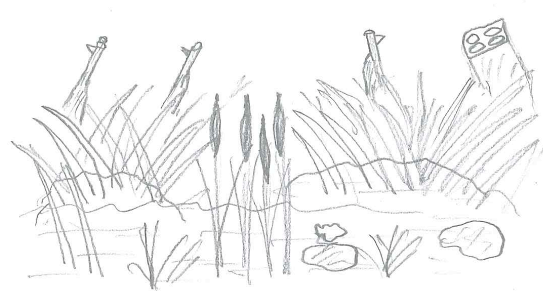
Montana Duck Hunting

Montana duck hunters are all perfect shots. Ten Montana hunters are in a duck blind when 10 ducks fly over. All 10 hunters pick a duck at random to shoot at, and all 10 hunters fire at the same time. How many ducks could be expected to escape?

For the calculation assume, as per tradition, the ducks are perfect spheres.





Answer of problem **Montana Duck Hunting**

We have D ducks and H hunters. Probability that a particular duck will not be hit is equal to probability that all hunters are aiming at other ducks. The probability that a hunter is aiming at some specific duck is $1/D$ and thus the probability of not being hit for a duck is

$$P_{\text{not hit}} = \left(1 - \frac{1}{D}\right)^H$$

and the average number of ducks to survive the ordeal is

$$N_{escape} = DP_{\text{not hit}} = D\left(1 - \frac{1}{D}\right)^H = \boxed{\frac{(D-1)^H}{D^{H-1}}} \approx 3.49 \qquad \text{for } D = H = 10$$

Another solution. Let's arbitrarily order the hunters. The first hunter will shoot a duck with probability 1; the second one will shoot a duck only if the duck he is aiming at has not been shot by the first hunter, the probability of this is $P = 1 - 1/D$. then the second hunter on average will kill P ducks. The third hunter will kill a duck only if he aims at a duck not hit by first or second hunters already, which has probability P^2, and so on. As a result the average number of killed ducks will be

$$K = 1 + P + \cdots + P^{H-1} = \frac{1 - P^H}{1 - P} = D(1 - P^H)$$

- always less or equal to D, independently of the hunters's number. The number of escaped ducks is

$$\boxed{N_{escae} = D - K = D\,P^H = D\left(1 - \frac{1}{D}\right)^H}$$