Sharing a Birthday

Take a class with N students in it. What is the probability that at least two of them have the same birthday? In class of what size you will be able to claim with 50% certainity that some students share a birthday? With 80% certainity?

(Assume a year has 365 days, all birthdays are equally probable, and after writing down the exact expression, derive an approximate formula for $N \ll 365$. It turns out this approximation works pretty well for all relevant N.)

Compare this result with probability that someone out of N students have the same birthday as *you*.

Answer of problem **Sharing a Birthday**

Let's calculate the probability that there are no students sharing a birthday. This means that if a student can have birthday falling on any of 365 days, a second student can only have 364 days available, a third student have 363 days available, and so on. For N students not to share a birthday the probability is,

$$\bar{P}_N = 1 \times \frac{364}{365} \times \frac{363}{365} \times \cdots \times \frac{365-(N-1)}{365} = \prod_{n=0}^{N-1} \left(1 - \frac{n}{365}\right)$$

This is the exact expression. To calculate the product approximately: take log, expand each term to first order and sum arithmetic series,

$$\ln \bar{P}_N = \sum_{n=0}^{N-1} \ln\left(1 - \frac{n}{365}\right) \approx -\sum_{n=0}^{N-1} \frac{n}{365} = -\frac{N(N-1)}{2 \cdot 365}$$

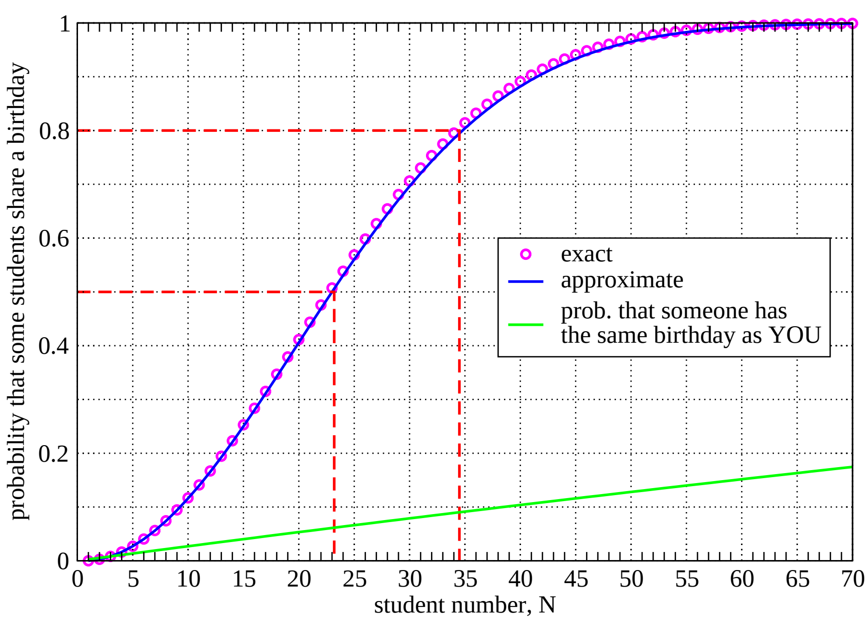
So for probability that some out of N students share a birthday is

$$\boxed{P_N = 1 - \bar{P}_N \approx 1 - e^{-\frac{N(N-1)}{2 \cdot 365}}}$$

Exponent is important! Just the first two terms in the expansion would not be enough!

Both exact and approximate expressions are plotted in the figure, and are almost on top of each other. To be able to claim with given certainity that some students share a birthday, one needs

$$\boxed{50\% \quad N = 23 \ students \qquad\qquad 80\% \quad N = 35 \ students}$$



The probability that some students might have the same birthday as you is $P_{you} = 1 - \bar{P}_{you}$, with probability that no students have the same birthday as you being,

$$\bar{P}_{you} = \left(\frac{364}{365}\right)^N = \left(1 - \frac{1}{365}\right)^N \approx e^{-N/365} \qquad \boxed{P_{you} = 1 - e^{-N/365}}$$