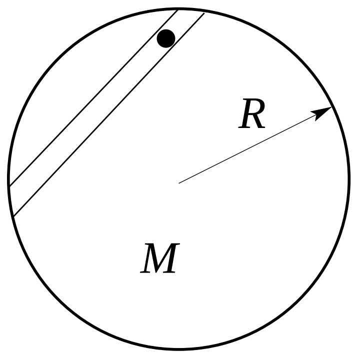
Wormhole travel



A straight tube is drilled between two points (not necessarily diametrically opposite) on the earth. An object is dropped into the tube. How much time does it take to reach the other end? Ignore friction, and assume (erroneously) that the density of the earth is constant.

Answer of problem **Wormhole travel**

Let the earths mass be M, and let its radius be R. Consider the object when it is a distance x from the point closest to the center of the earth, at radius r.

The gravitational force on the object is due to the mass of the earth that is inside the radius r. Since mass is proportional to volume, the mass inside the radius r is $M(r/R)^3$. The force on the object is therefore

$$F = \frac{GM(r/R)^3 m}{r^2} = \frac{GMmr}{R^3}$$

We are interested in the component of the force along the tube, which is

$$F_x = -F \sin\theta = -\frac{GMm}{R^3}x$$

Therefore, $F = ma$ along the tube gives

$$m\ddot{x} = -\frac{GMm}{R^3}x$$

This equation describes simple harmonic motion, with frequency

$$\omega = \sqrt{\frac{GM}{R^3}}$$

Therefore, the roundtrip time is

$$T = 2\pi/\omega = 84 \; minutes$$

The one-way time to the other end of the tube is therefore 42 minutes. Thats quick! Note that this result is independent of where chord is. The chord can be a diameter of the earth, or it can be a straight tube spanning the 20-foot width of a room.

Incidentally, the period T is also the time it takes to free-fall orbit the Earth right at the surface - what a coincidence!