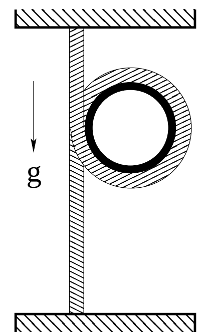
Falling emptiness



A uniform heavy rope, with ends fixed one above the other, tightly wraps around a massless ring. If the ring is let go, what is the acceleration of its fall?

Think about what would happen if the ring wasn't massless.

Answer of problem **Falling emptiness**

$$\boxed{a = \frac{g}{2}}$$

Solution:

the moving part here is the rope around the ring, rest of the rope is not moving at any given moment. This motion is that of a rolling wheel without sliding (one can imagine the ring rolling down if there is friction between the rope and the ring - and if there is no friction the rope just slides along the ring but the motion of the rope is still the same; you can convince yourself by looking at the motion of parts of the rope around the ring as it goes down, and comparing to a rolling motion). Consider the rotation around the instanteneous axis (point of the ring, that is not moving at a given moment - next to the vertical part of the rope). The angular acceleration around this axis is:

$$\alpha = \frac{\tau}{I}$$

with moment of inertia around this point

$$I = MR^2 + MR^2 = 2MR^2$$

and the torque

$$\tau = MgR$$

where $M = 2\pi R\rho$ is the mass of the rope around the ring. Thus we get,

$$\alpha = \frac{g}{2R} \qquad \Rightarrow \qquad a = \frac{g}{2}$$

The massive m ring problem I leave as an exercise. But now it will be important whether there is friction between the ring and the rope.