Problem of the Week

The bug climber

A stick of length $L$ is placed vertically by the wall. At its lower end sits a bug. The end B of the stick starts moving to the right with speed $v$, and at the same moment the bug starts crawling along the stick with speed $u$ relative to the stick. What is the maximal height above the floor that the bug reaches while it crawls along the stick? End A of the stick does not lose contact with the wall.
At time $t$, lower end of the stick will be distance $vt$ from the wall, making angle $\cos \alpha = vt/L$ with the floor, and the bug progressed to a point $ut$ from this end (along the hypotenuse). The height above the floor will be

$$h(t) = ut \sin \alpha = ut \sqrt{1 - \frac{v^2 t^2}{L^2}}$$

Maximal height will be reached at time $t = \frac{L}{v\sqrt{2}}$, when the stick makes angle 45 deg with the floor, and it will be

$$h_{\text{max}} = \frac{Lu}{2v}$$

If the bug is fast, and it reaches the end of the stick (in time $t = L/u$) before the stick makes 45-angle, then the maximal height is

$$h_{\text{max}} = L \sqrt{1 - \frac{v^2}{u^2}}$$