Capacitors

Use symmetries of the configurations to calculate their total capacitances.
(a) upper and lower branches are equivalent, so voltage across central capacitor is zero - throw it away.

\[ C_a = 2 \left( \frac{1}{C} + \frac{1}{2C} \right)^{-1} = \frac{4}{3} C \]

(b) From symmetry, voltage drops on outside C-capacitors are the same and we’ll denote them \( U_1 \), and similarly for \( 2C \) capacitors, \( U_2 \), with condition \( U_1 + U_2 = V \) - overall voltage drop. Voltage drop across middle one is \( U_2 - U_1 \). Write system of equations for charges on the capacitors, \( q_1 = CU_1, q_2 = 2CU_2 \) and \( q_{\text{center}} = q_1 - q_2 = C(U_2 - U_1) \) and solve this system for \( U_{1,2} \) and \( q_{1,2} \). The overall capacitance is

\[ C_b = \frac{q_1 + q_2}{V} = \frac{7}{5} C \]

(c) make a vertical cut after first vertical capacitor - resulting infinite chain is the same as the original

\[ \frac{1}{C_c} = \frac{1}{C} + \frac{1}{C + C_c} \quad \Rightarrow \quad C_c = \frac{\sqrt{5} - 1}{2} C \]

(d) the points with the same voltages can be connected - this results in equivalent scheme: (3 parallel)-(6 parallel)-(3 parallel)

\[ \frac{1}{C_d} = \frac{1}{3C} + \frac{1}{6C} + \frac{1}{3C} = \frac{5}{6C} \quad \Rightarrow \quad C_d = \frac{6}{5} C \]