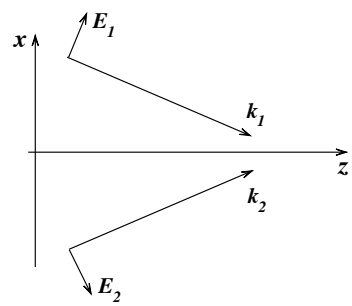
E&M II - set 9 :: Waveguides

PHSX 520 - Fall 2015

Problem 1

One can understand waves in a waveguide by looking at superposition of several plane waves. The superposition produces inhomogeneous waves.

Electromagnetic wave represents a superposition of two monochromatic uniform plane waves with the same frequency and amplitude, and phase (in vacuum). Vectors of the electric field in both waves are in the plane formed by the wave vectors $\mathbf{k}_1 = (-k_x, 0, k_z)$ and $\mathbf{k}_2 = (k_x, 0, k_z)$ and directed as shown in the figure.



(a) Find the expressions for the total electric and magnetic fields in the wave. Is it a TE or TM wave?

(b) Now assume that the region of the wave is limited by two infinite well-conducting flat planes placed at $x = \pm a/2$ (the wave exists in $-a/2 < x < a/2$). What values we can have for k_x in such a wave? What is the minimal frequency needed to have a wave propagating along z?

(c) In the xz plane sketch the physical field $\mathbf{E}(x, z)$ for smallest possible k_x (for one period along z at $t = 0$). On the same plot indicate direction and amplitude of magnetic field in the wave (dots or crosses indicate direction and their density indicate the amplitude).

Problem 2

Find the eigenvalues for $\gamma = \sqrt{\omega^2/c^2 - k_z^2}$ and corresponding eigenmodes for TE waves in a cylindrical waveguide with radius a. Write down expressions for all components of E and B fields. $\varepsilon = \mu = 1$.

Problem 3

Consider a rectangular waveguide $a \times b$. The waveguide is excited through a small opening at $z = 0$ end, where the incoming field is given by $\mathbf{E}(x, y, t) = \hat{\mathbf{z}} E_0 \, ab\delta(x - a/4)\delta(y - b/2)\, e^{-i\omega t}$ (very localized source). Find the weights of different TM waves excited by such source. What is the lowest mode that has maximal weight?