E&M II - set 8 :: Diffraction

PHSX 520 - Fall 2015

Problem 1

Using the scalar diffraction theory (Kirchhoff integral) calculate and sketch the diffraction pattern of a plane wave normally incident on an infinite flat screen with

(a) one vertical slit of width $2a$;

(b) a large number N of vertical slits of width $2a$, separated by distance $2d$. For the sketch take $d/a = 5$.

Assume weak deviation from the geometrical optics, $ka \gg 1$, and show that the diffraction pattern in this case is one-dimensional (function of angle perpendicular to the slit).

Problem 2

Poisson-Arago spot.
Consider a flat non-transparent disk or radius a. A point source of light ψ_0 lies on the axis going through the center of the disk (reasonably far, at a point $-z$). Show that the intensity of light at the mirror point is given by

$$I = \frac{I_0}{4} \frac{z^2}{a^2 + z^2}$$

where $I_0 = (\psi_0/\sqrt{a^2 + z^2})^2$ is the intensity at the rim of the disk. Use the geometric optics limit, $ka \gg 1$.

Hint: write down the general expression for the field at a spot z_1 on the other side of the screen, using approximation $zk, z_1 k \gg 1$; and in the radial integral introduce a new variable $\rho = R(z) + R(z_1)$ where R's are distances from an aperture point to points z, z_1. In evaluation of the integral (by parts) leave only the lowest order term in $1/(k\rho)$.

This means that some distance away from the screen ($z \gg a$) in the region of geometrical shadow there will be a sufficiently bright spot on the symmetry axis. Poisson predicted this spot in 1818 using Fresnel's theory to demonstrate that the wave hypothesis of light was wrong. However, Arago went ahead and found this spot experimentally!