E&M II - set 6 :: Scattering

PHSX 520 - Fall 2015

Problem 1

A transverse plane wave incident on a small sphere of radius a ($a \ll \lambda$) made of a perfect conductor. (Perfect conductor means that the conductivity is so large, that the skin depth $\delta = \sqrt{c^2/2\pi\sigma\omega}$ is very small even in the static limit, i.e. even for $\omega \to 0$ we have $\omega\sigma \to \infty$, and thus both electric and magnetic fields inside can be taken zero: $\mathbf{E} = \mathbf{D} = 0$, $\mathbf{H} = \mathbf{B} = 0$.)

(a) Assuming the electric field of the wave is uniform near the sphere, find the electric dipole induced on the sphere.

(b) Find the magnetic dipole induced on the sphere by the magnetic field of the wave.

Hint: use the corresponding scalar potentials for both electric and magnetic fields, and apply appropriate boundary conditions at $r = a$.

(c) Find the differential scattering cross section. List the values for scattering between all different (linear) polarizations.

(d) Find the differential cross section for unpolarized light; the complete differential cross section; the polarization $\Pi(\theta)$ and the total cross section of scattering. Find the angle of maximum polarization of scattered light, and the angle of maximal scattering.

(e) Now assume we have an array of N such scatterers, stretched along a single straight line with distance ℓ between two neighboring spheres. Calculate the additional angular dependence of scattering given by the structure factor $F(\theta)$ when the array is oriented along and perpendicular to the direction of the incoming wave. Analyse this angular dependence for the case when $\ell \ll \lambda$, but the entire array size is $L = \ell N \gg \lambda$.

Problem 2

Consider scattering of a left-circularly-polarized plane wave (incoming along the z-axis) on a weak dielectric in a form of a very thin wire of length L pointing in direction $\boldsymbol{\ell} = (\theta_0, \phi_0 = 0)$.

(a) Find the differential cross-section using perturbation theory between waves with general initial and final polarizations.

(b) Determine scattering direction along which the light is the most left-circularly polarized.