E&M II - set 3 :: Reflection & Refraction

PHSX 520 - Fall 2015

Problem 1

Show that if a straight, transparent pipe is to act as an optical fibre for any light ray incident on one end, its index of refraction (relative to air) must satisfy $n^2 > 2$.

Problem 2

Show that the time averaged flow of radiation into a thick slab of metal is equal to the time averaged Joule heating. Assume a good conductor, $\sigma \gg \omega$.

Problem 3

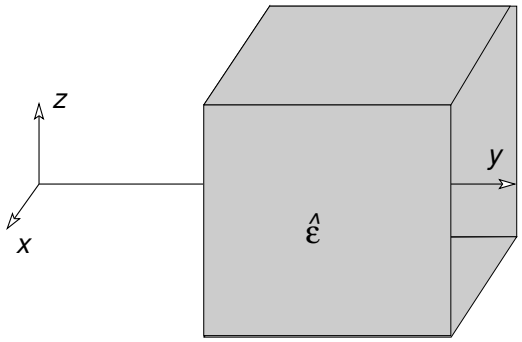
Find and plot the dispersion characteristics (dependence of ω on k in a wave, as in $\omega(k) = ck$) for transverse ($\mathbf{E} \perp \mathbf{H} \perp \mathbf{k}$) and longitudinal ($\mathbf{E} \parallel \mathbf{k}$) waves in plasma using constitive equation $\mathbf{D} = \varepsilon(\omega)\mathbf{E} + \delta^2(\omega)\,\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E})$, that takes into account thermal motion of particles. Here $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$ and $\delta(\omega) = v_T/\omega$, where v_T is the average thermal speed of electrons.

Problem 4

A plane wave is incident on a dielectric material, from the vacuum, $\omega = ck$, on the left. The dielectric has $\mu = 1$ and complex dielectric tensor oriented as follows

$$\epsilon_{ij} = \begin{pmatrix} \alpha & i\beta & 0 \\ -i\beta & \alpha & 0 \\ 0 & 0 & \gamma \end{pmatrix}$$

where α, β, γ are real numbers.



(a) Write out the electric field everywhere if the incident wave is

$$\mathbf{E}_i(\mathbf{r}, t) = E_0 \hat{x} e^{i(ky - \omega t)}$$

(b) Repeat part (a) if the incident wave is

$$\mathbf{E}_i(\mathbf{r}, t) = E_0 \frac{\hat{z} + i\hat{x}}{\sqrt{2}} e^{i(ky - \omega t)}$$