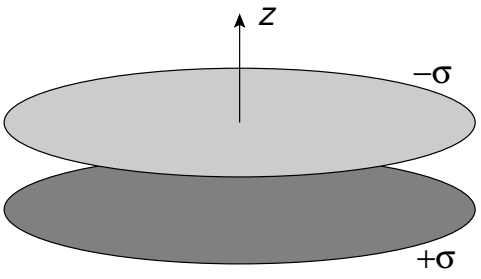
E&M II - Time-dependent EM potentials

PHSX 520 - Fall 2015

Problem 1

A circular-plate capacitor is charged with $-\sigma_0 t/\tau$ surface density on the upper plate, and $+\sigma_0 t/\tau$ on the lower plate. The charge on the plates grows linear in time, and τ is a time constant. Assume the radius of the capacitor is large compared to the plate separation d, the surface charge is uniform, and $c\tau \gg d$. Ignore the edge effects.



(a) Find the electric and magnetic fields between the plates

(b) Find the electromagnetic potentials in Coulomb gauge

(c) Find the electromagnetic potentials in $\varphi = 0$ gauge

Problem 2

Show, by direct differentiation, that the retarded potentials

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{c}\int \frac{\mathbf{j}(\mathbf{r}', t - |\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}\, dV' \,, \qquad \varphi(\mathbf{r},t) = \int \frac{\boldsymbol{\rho}(\mathbf{r}', t - |\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|}\, dV'$$

satisfy Lorenz gauge

$$\boldsymbol{\nabla}\cdot\mathbf{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0\,.$$

Assume that all relevant integrals are convergent.